JOURNAL OF URBAN ECONOMICS **40,** 279–302 (1996)
ARTICLE NO. 0033

Second-Best Congestion Pricing: The Case of an Untolled Alternative*

ERIK VERHOEF,† PETER NIJKAMP, AND PIET RIETVELD

Department of Spatial Economics, Free University, De Boelelaan 1105, 1081 HV, Amsterdam, The Netherlands

Received October 14, 1994; revised March 15, 1995

This paper deals with second-best one-route congestion pricing in case of an untolled alternative. Using a two-link network simulation model, the effects of various demand and cost parameters on the relative efficiency of one-route tolling are analyzed. It is investigated whether the existence of costs of congestion charging may be a reason for one-route tolling to be more "overall efficient" than two-route tolling. Finally, the efficiency of revenue-maximizing one-route and two-route tolling is discussed.

1. INTRODUCTION

In this paper we study the relative efficiency of second-best congestion pricing in the case where road users can choose between a tolled and an untolled route. Clearly, as people will generally prefer free (or at least cheap) alternatives, the resulting choice processes are particularly interesting when considering congestion. There are various reasons why such situations may occur in practice. First, the regulator may leave an alternative untolled for equity reasons; for instance to protect low-income groups from having to pay fees, or to increase the social feasibility of road pricing (see Starkie [15]). Alternatively, untolled alternatives may be present when (electronic) toll experiments are being undertaken. Furthermore, the same type of situation prevails with the occurrence of so-called "rat-running:" drivers using escape routes in order to avoid certain toll-points. On the other hand, the cost of toll collection may actually justify the choice of not regulating an entire road network, but only some of its major links instead. Finally, part of the road infrastructure may be privately owned and tolled, with a publicly provided alternative offered for free. In what follows, the efficiency sides to one-route tolling are discussed, and we are therefore

* The authors would like to thank Kenneth Button, Richard Emmerink, and two anonymous referees for very useful comments on an earlier version of this paper.

† Erik Verhoef is affiliated to the Tinbergen Institute and participates in the VSB-fonds sponsored research project "Transport and Environment."

0094-1190/96 $18.00

concerned with the latter four types of reason for untolled alternatives to exist.

Probably the first (qualitative) discussion of the problem of congestion regulation on two routes can be found in the classic article by Knight [10]. Later on, Lévy-Lambert [11] and Marchand [12] were the first to derive the optimal one-route toll with an untolled alternative. Recent discussions of two-route problems in the context of the dynamic bottleneck model with inelastic demand can be found in Arnott *et al.* [2, 5] and Bernstein and El Sanhouri [6] (although the latter actually also do consider elastic demand, but not with the second-best optimal one-route toll). In our static equilibrium approach, elasticity of demand, for instance resulting from the presence of alternative transport modes, can easily be considered and will actually turn out to be of crucial importance for the efficiency of one-route tolling. On the other hand, dynamic departure time decisions will be ignored, which renders our analyses supplementary to the latter three mentioned above. In addition, we will also pay attention to (private) revenue maximizing tolling.

Other recent writings on second-best regulation of road transport externalities include Verhoef *et al.* [17] on the efficiency of non-differentiated tolling, Wilson [22] and d'Ouville and McDonald [13] on optimal road capacity supply with suboptimal congestion pricing, Braid [7] and Arnott *et al.* [3] on uniform or stepwise pricing of a bottleneck, Arnott [1], Sullivan [16], and Fujita [8] on congestion policies through urban land use policies, and Arnott *et al.* [4], Glazer and Niskanen [9], and Verhoef *et al.* [18] on regulatory parking policies.

The plan of the paper is as follows. Section 2 starts off by investigating the optimal one-route toll and its welfare economic properties. By comparing these to first-best regulation, we are able to evaluate the relative performance of one-route tolling in Section 3. In addition, we will discuss the question of whether the cost of congestion charging itself may be a reason for leaving some alternatives untolled. In Section 4 we consider the case where tolling occurs for the purpose of revenue raising by some private operator. Section 5 contains the conclusions.

2. OPTIMAL SECOND-BEST CONGESTION PRICING WITH AN UNTOLLED ALTERNATIVE: SOME BASIC WELFARE ECONOMIC PROPERTIES

In this section we discuss some basic welfare economic properties of congestion pricing with an untolled alternative. We study a simple network with two competing, (possibly) congested routes, one tolled (route T), and one untolled (route U). It is assumed that the regulator wishes to set the fee on the tolled route so as to maximize efficiency under the inherent limitation of not tolling the other route. In doing so, some sub-goals related to overall efficiency have to be traded off. These are (1) an overall demand (‘‘modal split’’) effect, being the extent to which road users efficiently leave the road system altogether due to congestion pricing, and (2) a route split effect, being the extent to which the remaining road users divide themselves efficiently among both routes. Generally, as one of the routes remains untolled, it will be impossible to realize the first-best situation where both effects are optimized (see also Bernstein and El Sanhouri [6]).

When considering congestion pricing with an untolled alternative, one has to take account of both demand and cost interdependencies between the two routes. In the problem’s most pure form, the public regards the two alternative routes as perfect substitutes. We therefore consider one single demand function $D(N)$, where N denotes the total number of road users (on both routes), and two average user cost functions $c_{\mathrm{T}}(N_{\mathrm{T}})$ and $c_{\mathrm{U}}(N_{\mathrm{U}})$, where naturally $N = N_{\mathrm{T}} + N_{\mathrm{U}}$ and where average user cost and also the value of time are assumed to be equal for all road users. In line with Wardrop’s first principle (Wardrop [21]), at any equilibrium the average cost on route U should then be equal to the average cost on route T plus the one-route fee f; otherwise people would shift from the one route to the other. Furthermore, both should be equal to marginal benefits $D(N)$. The optimal one-route toll in this setting has originally been derived by Lévy-Lambert [11] and Marchand [12]. The reason for presenting our alternative derivation is that we think it is significantly more transparent and easy to follow than the two above mentioned.

Assuming that the regulator aims at maximizing total benefits, as given by the area under the demand curve, minus total costs, he has to solve the following Lagrangian:

$$\begin{aligned}\mathscr{L} = \int_0^N D(n)\,dn - N_{\mathrm{T}}\cdot c_{\mathrm{T}}(N_{\mathrm{T}}) - N_{\mathrm{U}}\cdot c_{\mathrm{U}}(N_{\mathrm{U}}) \\ + \lambda_{\mathrm{T}}\cdot\big(D(N) - c_{\mathrm{T}}(N_{\mathrm{T}}) - f\big) + \lambda_{\mathrm{U}}\cdot\big(D(N) - c_{\mathrm{U}}(N_{\mathrm{U}})\big) \qquad (1)\end{aligned}$$

with $N = N_{\mathrm{T}} + N_{\mathrm{U}}$. The first-order conditions are

$$\begin{aligned}\frac{\partial\mathscr{L}}{\partial N_{\mathrm{T}}} &= D(N) - c_{\mathrm{T}}(N_{\mathrm{T}}) - N_{\mathrm{T}}\cdot c'_{\mathrm{T}}(N_{\mathrm{T}}) + \lambda_{\mathrm{T}}\cdot\big(D'(N) - c'_{\mathrm{T}}(N_{\mathrm{T}})\big) \\ &\quad + \lambda_{\mathrm{U}}\cdot D'(N) = 0 \\ \frac{\partial\mathscr{L}}{\partial N_{\mathrm{U}}} &= D(N) - c_{\mathrm{U}}(N_{\mathrm{U}}) - N_{\mathrm{U}}\cdot c'_{\mathrm{U}}(N_{\mathrm{U}}) + \lambda_{\mathrm{T}}\cdot D'(N) \\ &\quad + \lambda_{\mathrm{U}}\cdot\big(D'(N) - c'_{\mathrm{U}}(N_{\mathrm{U}})\big) = 0\end{aligned}$$

$$\frac{\partial \mathscr{L}}{\partial f} = -\lambda_{\mathrm{T}} = 0$$

$$\frac{\partial \mathscr{L}}{\partial \lambda_{\mathrm{T}}} = D(N) - c_{\mathrm{T}}(N_{\mathrm{T}}) - f = 0$$

$$\frac{\partial \mathscr{L}}{\partial \lambda_{\mathrm{U}}} = D(N) - c_{\mathrm{U}}(N_{\mathrm{U}}) = 0.$$

Using $\lambda_{\mathrm{T}} = 0$, we find

$$f = N_{\mathrm{T}} \cdot c'_{\mathrm{T}}(N_{\mathrm{T}}) - \lambda_{\mathrm{U}} \cdot D'(N).$$

Solving for λ_{U} yields

$$\lambda_{\mathrm{U}} = \frac{N_{\mathrm{U}} \cdot c'_{\mathrm{U}}(N_{\mathrm{U}})}{D'(N) - c'_{\mathrm{U}}(N_{\mathrm{U}})}.$$

Substitution of λ_{U} then yields the optimal second-best fee

$$f = N_{\mathrm{T}} \cdot c'_{\mathrm{T}}(N_{\mathrm{T}}) - N_{\mathrm{U}} \cdot c'_{\mathrm{U}}(N_{\mathrm{U}}) \cdot \left(\frac{-D'(N)}{c'_{\mathrm{U}}(N_{\mathrm{U}}) - D'(N)} \right), \tag{2}$$

while it can be shown that the first-best road price r_i on route i ($i = \mathrm{T}, \mathrm{U}$) would be

$$r_i = N_i \cdot c'_i(N_i). \tag{3}$$

The first term in (2), equal to the marginal external congestion costs on route T in the second-best optimum, captures the direct impact of the fee on congestion on the tolled route itself. However, the second term indicates that, for optimal use of the fee, one should also take account of the ‘‘spill-over’’ effects on the untolled route by subtracting some non-negative term, which is a fraction of the marginal external congestion costs on the untolled route in the second-best optimum. This fraction (which may range between 0 and 1) is given by the term between the large parentheses and depends on the (relative) values of $D'(N)$ and $c'_{\mathrm{U}}(N_{\mathrm{U}})$; that is, on the slopes of the demand curve and the average cost curve on the untolled route in the second-best optimum.

The actual impact of D' and c'_{U} on the expression for f is a bit hard to trace at once, as both appear twice in the second term. However, we can obtain some insight by considering some extreme values of D' and c'_{U}. For instance, if overall demand is perfectly inelastic ($D' = -\infty$) in the second-best optimum, the term between the large parentheses in (2) approaches 1 and (2) reduces to

$$f = N_{\mathrm{T}} \cdot c'_{\mathrm{T}}(N_{\mathrm{T}}) - N_{\mathrm{U}} \cdot c'_{\mathrm{U}}(N_{\mathrm{U}}). \tag{2'}$$

As there is no effect of the policy on overall demand, but solely on route split, the best thing the regulator can do is to concentrate on achieving the optimal route split. Hence, the fee should be set at the difference between the marginal external congestion costs on both routes in the second-best optimum, in order to attain the efficient distribution of road users over both routes. Note that, particularly when approaching this extreme case, one cannot tell in advance whether (2) yields a positive tax. It may well be negative, implying an optimal subsidy on using route T. In the extreme case of completely inelastic overall demand, this would be the case whenever marginal external congestion costs are higher on the untolled route than on the tolled route in the second-best optimum. Another possibility in this case would of course be taxation of route U, leaving T untolled.

Alternatively, if overall demand is perfectly elastic $(D' = 0)$ in the second-best optimum, (2) reduces to the following extreme expression:

$$f = N_{\mathrm{T}} \cdot c'_{\mathrm{T}}(N_{\mathrm{T}}). \tag{2''}$$

The regulator may now ignore spill-over effects from route T to route U since road usage on route U remains unaffected in any way: due to the completely homogeneous and sufficiently large group of potential road users, they will keep on entering route U up to a level where (constant) marginal benefits are equal to marginal private cost, regardless of the type of regulation on route T. Therefore, the best thing the regulator can do in this case is to optimize usage of route T, ignoring the unavoidable welfare loss on route U. In contrast to the former case, overall demand effects of regulation (be it solely on route T), rather than route split impacts, now receive full attention.

The same expression (2″) for the optimal one-route toll is found in the case considered by Knight [10], where route U is complete uncongested ($c'_{\mathrm{U}} = 0$) in the second-best optimum. Since regulation on route T then apparently causes no spill-over cost in terms of increasing congestion externalities on route U (there *is* no congestion on route U in the second-best optimum), the regulator may just optimize road use on route T as if first-best conditions would apply.

We will not discuss the fourth extreme case, where route U is completely congested in the second-best optimum ($c'_{\mathrm{U}} = \infty$), as this is of course a highly unrealistic one. It is hard to image how, given the extreme congestion on route U, the average user costs on route U and route T can still be equal, as they should be in such an equilibrium. It is perhaps worth underlining here that road usage is not measured in flows (in which case, because of the backward-bending cost curve, its derivative will indeed be infinite at some relevant point), but in numbers of road users instead.

Generally then, we may conclude that the second-best one-route congestion fee in its optimal use trades off a number of "sub-goals" contributing to the overall goal of efficient allocation. These sub-goals are related to usage and congestion on both routes, and therefore comprise overall demand and route split effects.

Figure 1 gives a diagrammatic sketch of the situation discussed above. In the right panel, the demand (D), marginal private cost (MPC), and marginal social cost (MSC) curves are drawn for the entire group of road users using two identical alternatives U and T (we use identical routes for ease of diagrammatic presentation). The middle and left panels give the cost curves for both routes. Optimal first-best regulation implies a levy r on both routes, leading to a reduction in usage from N^0 to N^r, composed of reductions from N_{T}^0 and N_{T}^r and from N_{U}^0 to N_{U}^r. The increase in social welfare is given by the surface of the bold triangle in the right panel.

However, the use of the optimal second-best fee f on route T alone, as given by (2), leads to a reduction in the number of trips made on route T from N_{T}^0 to N_{T}^f, and an *increase* in the number of trips made on route U from N_{U}^0 to N_{U}^f because of route switching (note that switching occurs so

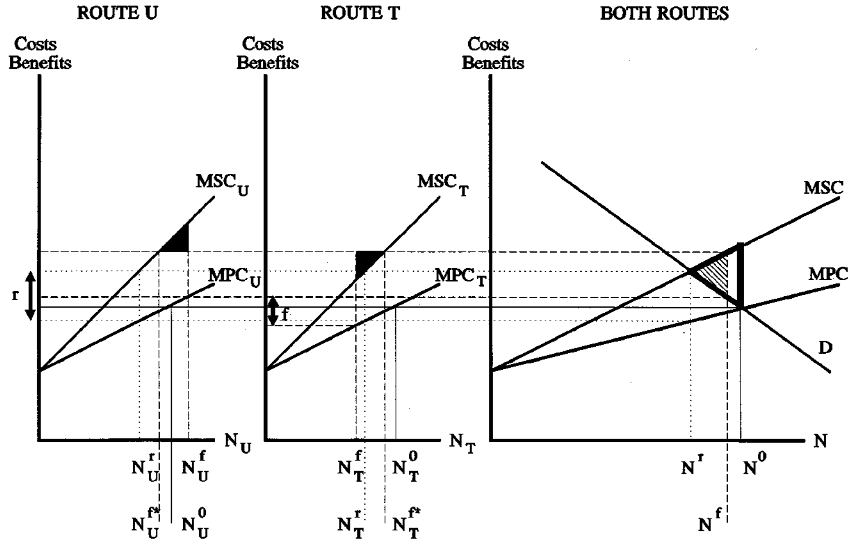

FIG. 1. The welfare effects of first-best two-route (bold) versus second-best one-route (bold minus shaded and black) congestion pricing.

that marginal benefits on both routes remain equalized). The reduction in total usage is therefore only from N^0 to N^f, and the use of this policy misses out on the potential welfare gains as given by the shaded triangle in the right panel. Moreover, the reduction from N^0 to N^f is not accomplished in its most efficient way, which would have been reductions to N_{T}^{f*} and N_{U}^{f*}, respectively (where the marginal social costs on both routes are equalized). Therefore, the two black triangles give additional welfare losses of one-route tolling in comparison with first-best regulation. Obviously, the total welfare gain of optimal one-route tolling will be non-negative; otherwise the optimal second-best fee would simply be zero.

In Verhoef *et al.* [17], an index of relative welfare improvement ω was used, defined as the ratio of the overall welfare gain under second-best regulation compared to non-intervention, and the overall welfare gain under first-best regulation compared to non-intervention. This index will in this case be

$$\omega = \left[\int_{N^f}^{N^0} S(n)\,dn - \int_{N_{\mathrm{U}}^{f*}}^{N_{\mathrm{U}}^{f}} \left(k_{\mathrm{U}}(n_{\mathrm{U}}) - k_{\mathrm{U}}\left(N_{\mathrm{U}}^{f*}\right)\right) dn_{\mathrm{U}} - \int_{N_{\mathrm{T}}^{f}}^{N_{\mathrm{T}}^{f*}} \left(k_{\mathrm{T}}\left(N_{\mathrm{T}}^{f*}\right) - k_{\mathrm{T}}(n_{\mathrm{T}})\right) dn_{\mathrm{T}}\right] \Bigg/ \int_{N^r}^{N^f} S(n)\,dn + \int_{N^f}^{N^0} S(n)\,dn, \tag{4}$$

where the function S is defined as the difference between marginal social cost minus marginal benefits and k_i denotes marginal social cost for group i. The denominator of (4) represents the bold triangle in the right panel of Fig. 1. The first term in the numerator gives the surface of this triangle minus the shaded area, and the second and third term represent the two black triangles in the left and middle panel, respectively. We will consider the various factors determining the value of ω in the next section.

3. FACTORS DETERMINING THE RELATIVE PERFORMANCE OF ONE-ROUTE TOLLING

In this section we discuss the outcomes of some simulations that were performed in order to arrive at some more explicit results than the general specification in the foregoing section allows. When switching toward explicit functions, one is soon confronted with very tedious expressions, depending of course on the functional forms chosen for the demand and cost functions. As there is no theoretical reason to prefer any of the functional forms possible, and in order to keep the analysis manageable and the outcomes tractable, we decided to keep the simulation model as simple as possible by assuming that these functions are affine over the relevant ranges considered (that is, the range containing the non-intervention, second-best, and first-best levels of usage). Although the use of affine functions may be criticized, they are in any case sufficient to serve the general goal of the simulations, which is the assessment of the influence of some key factors related to demand and cost structures on the relative performance of one-route tolling. Finally, it is worth mentioning that Arnott *et al.* [2, 5] have pointed out at several occasions that the affine congestion cost function is not necessarily unreasonable, since it can be interpreted as a reduced form representation of the Vickrey [20] bottleneck model.

3.1. *The Model*

The model then consists of one joint affine demand function, characterized by slope α and intersection δ,

$$D = \delta - \alpha \cdot (N_{\mathrm{T}} + N_{\mathrm{U}}). \tag{5}$$

Next, for both routes ($i = \mathrm{T}, \mathrm{U}$), the marginal private cost MPC, equal to average social cost ASC, consists of a free-flow cost component κ_i and a congestion cost component which is assumed to be proportional to total usage N_i with a factor β_i,

$$\mathrm{MPC}_i = \mathrm{ASC}_i = \kappa_i + \beta_i \cdot N_i; \qquad i = \mathrm{T}, \mathrm{U}. \tag{6}$$

All parameters are non-negative, and we will only consider ‘‘regular’’ networks, where both routes are at least marginally used under non-intervention and under both types of regulation (that is, first-best and second-best). Apart from the explicit functions, the model is further identical to the one presented in Section 2. Under the three different regulatory regimes of non-intervention, second-best one-route tolling, and first-best two-route tolling, equilibrium usage on both routes will be as given in Table 1.

For the ‘‘base case’’ of our model, the following parameter values were chosen: $\alpha = 0.01$; $\delta = 50$; $\kappa_{\mathrm{T}} = \kappa_{\mathrm{U}} = 20$; and $\beta_{\mathrm{T}} = \beta_{\mathrm{U}} = 0.02$. So, both routes are assumed to be identical in the base case. No surprise then that equilibrium usage under non-intervention is equal on both routes: $N_{\mathrm{T}} = N_{\mathrm{U}} = 750$. Marginal private cost amounts to 35.00 on both routes; marginal social cost to 50.00. Under first-best regulation, optimal road prices of $r_{\mathrm{T}} = r_{\mathrm{U}} = 10.00$ are found for both routes, with marginal private cost then amounting to 30.00 and marginal social cost to 40.00 on both routes. The optimal road prices for both routes are therefore equal to the difference between these two, as theory dictates. Optimal usage is 500 on both routes. Under second-best one-route tolling, the second-best optimal fee for route T is 5.45. Marginal private cost is 30.91 on route T, and 36.36 on route U; the difference is exactly equal to the additional fee on route T, so that user equilibrium is indeed guaranteed. Marginal social cost is 41.82 on route T,

TABLE 1

Equilibrium Usage on Both Routes under Non-intervention, First-Best, and Second-Best Regulatory Policies

	Non-intervention	Two-route tolling	One-route tolling
N_{T}	$\dfrac{\delta - \dfrac{\alpha \cdot (\kappa_{\mathrm{T}} - \kappa_{\mathrm{U}})}{\beta_{\mathrm{U}}} - \kappa_{\mathrm{T}}}{\dfrac{\alpha \cdot \beta_{\mathrm{T}}}{\beta_{\mathrm{U}}} + \alpha + \beta_{\mathrm{T}}}$	$\dfrac{\delta - \dfrac{\alpha \cdot (\kappa_{\mathrm{T}} - \kappa_{\mathrm{U}})}{2 \cdot \beta_{\mathrm{U}}} - \kappa_{\mathrm{T}}}{\dfrac{\alpha \cdot \beta_{\mathrm{T}}}{\beta_{\mathrm{U}}} + \alpha + 2 \cdot \beta_{\mathrm{T}}}$	$\dfrac{\delta - \dfrac{\alpha \cdot \big(\alpha/(\alpha + \beta_{\mathrm{U}})\big) \cdot (\kappa_{\mathrm{T}} - \kappa_{\mathrm{U}})}{\big(1 + \alpha/(\alpha + \beta_{\mathrm{U}})\big) \cdot \beta_{\mathrm{U}}} - \kappa_{\mathrm{T}}}{\dfrac{\alpha \cdot \big(\alpha/(\alpha + \beta_{\mathrm{U}})\big) \cdot 2 \cdot \beta_{\mathrm{T}}}{\big(1 + \alpha/(\alpha + \beta_{\mathrm{U}})\big) \cdot \beta_{\mathrm{U}}} + \alpha + 2 \cdot \beta_{\mathrm{T}}}$
N_{U}	$\dfrac{\delta - \dfrac{\alpha \cdot (\kappa_{\mathrm{U}} - \kappa_{\mathrm{T}})}{\beta_{\mathrm{T}}} - \kappa_{\mathrm{U}}}{\dfrac{\alpha \cdot \beta_{\mathrm{U}}}{\beta_{\mathrm{T}}} + \alpha + \beta_{\mathrm{U}}}$	$\dfrac{\delta - \dfrac{\alpha \cdot (\kappa_{\mathrm{U}} - \kappa_{\mathrm{T}})}{2 \cdot \beta_{\mathrm{T}}} - \kappa_{\mathrm{U}}}{\dfrac{\alpha \cdot \beta_{\mathrm{U}}}{\beta_{\mathrm{T}}} + \alpha + 2 \cdot \beta_{\mathrm{U}}}$	$\dfrac{\delta - \dfrac{\alpha \cdot (\kappa_{\mathrm{U}} - \kappa_{\mathrm{T}})}{2 \cdot \beta_{\mathrm{T}}} - \kappa_{\mathrm{U}}}{\dfrac{\alpha \cdot \big(1 + \alpha/(\alpha + \beta_{\mathrm{U}})\big) \cdot \beta_{\mathrm{U}}}{2 \cdot \beta_{\mathrm{T}}} + \alpha + \beta_{\mathrm{U}}}$

and 52.73 on route U, readily demonstrating a non-optimal route split: road usage is 545 on route T and 818 on route U. Finally, the index of relative welfare improvement ω is equal to 0.273 in this base case, indicating that 27.3% of the potential efficiency gains under two-route tolling will be realized with one-route tolling.

By varying the model's respective parameters, it is possible to gain insight into the conditions under which one-route tolling is a relatively (un)attractive option. The results are presented below. Obviously, with this type of modelling exercise, there are many more possibilities than just the ones reported below. Many of these have been studied; the ones discussed below are those that were found to be the most interesting.

3.2. *Varying Cost Parameters*

First, we consider the free-flow cost parameters, for instance reflecting the length of the links. Figure 2 gives the course of ω, and of the optimal first-best and second-best fees, for an increasing difference between these parameters. Instead of varying just one of the parameters, we simultaneously raised κ_T while lowering κ_U, both by 1.5 each step, the base case of $\kappa_T = \kappa_U = 20$ being in the center. First of all, although the congestion parameters are equal for both routes, first-best tolls on both routes do not coincide. This may be surprising at first sight, as one might expect the level

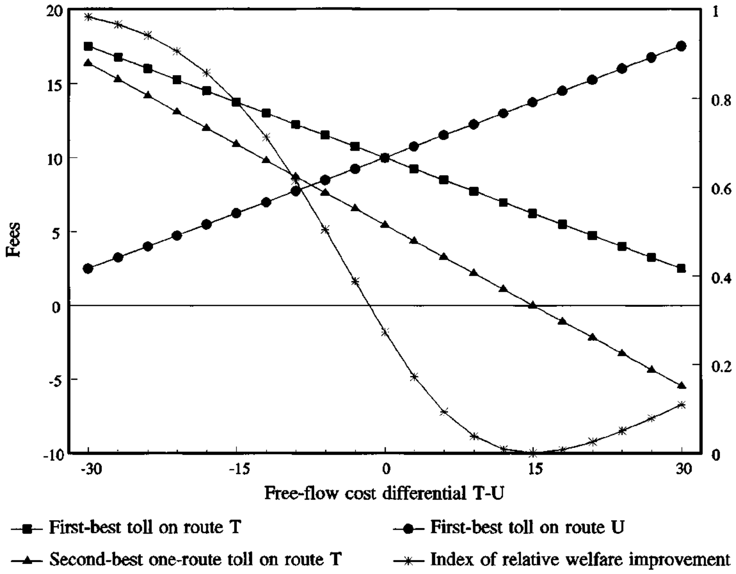

FIG. 2. Varying free-flow costs: optimal fees and index of relative welfare improvement.

of free-flow cost to be a purely ‘‘internal’’ cost component, without any impact on optimal fees. However, due to the fact that road users distribute themselves among the two routes such that marginal private cost (including the internal congestion cost component) are equalized, there is a direct effect of differences in free-flow costs on the optimal tolls. Generally speaking, the lower the free-flow cost, the higher the internal congestion cost, and hence also the higher the external congestion cost will be. This is illustrated in Fig. 2 by the courses of the first-best fees.

Due to this effect, also the optimal one-route toll and the index of relative welfare improvement show an interesting pattern in Fig. 2. When the free-flow costs on route T grow sufficiently high compared to those on route U (on the right-hand side of the figure), the optimal one-route toll may actually turn into a subsidy (f is negative). Marginal social costs on route U are then so much higher than those on route T that it is even worthwhile to attract some new traffic as a negative side-effect to the main aim of diverting traffic from route U to route T. For obvious reasons, first-best congestion tolls will never be negative. At the turning point, where f changes sign and is equal to zero, the index of relative welfare improvement therefore reaches its theoretical minimum of zero. Sticking to the range where the optimal one-route toll is a tax, it is clear that second-best regulation becomes more attractive the lower the free-flow costs on route T compared to those on route U. This is due to the fact that the regulator then controls that route which is, in the first place, more important in terms of usage and, secondly, where market forces tend to give rise to the largest congestion externalities. The first of these two reasons is illustrated by the fact that the optimal one-route toll approaches the optimal first-best toll on route T in these cases.

Also for varying differences in the congestion cost parameters β_i while keeping $\kappa_T = \kappa_U = 20$ (not presented graphically here, but the interested reader is referred to Verhoef *et al.* [19]), one-route tolling is more efficient, the higher β_U is in comparison to β_T. Apart from the fact that the regulator then again controls the more important route, route U then becomes an increasingly unattractive alternative for route T because of the internal part of the congestion cost, which makes the occurrence of adverse spill-overs due to regulation on route T less likely. In contrast to Fig. 2, for any difference in the congestion cost parameters, the first-best fees will be equal for both routes. The reason for this again perhaps counter-intuitive result becomes clear after considering the equilibrating effects of user behavior. In the first-best optimum, marginal social cost should be equalized between the two routes. Given the fact that road users distribute themselves over both links such that average social cost are equalized, and given the equality of free-flow costs and the affine form of the marginal cost functions, the result follows. Finally, with equal free-flow costs, f will not fall below zero.

In conclusion, for both types of cost parameter, the regulator should preferably perform one-route tolling on the lower cost route.

3.3. *Varying Demand Characteristics*

Next, we consider the demand parameters. Figure 3 shows what happens when ‘‘tilting’’ the demand curve around the original non-intervention equilibrium, doubling the slope at each step. Both the slope α and intersection δ therefore change simultaneously, in order to avoid ending up with very large (small) markets when demand approaches perfect (in-)elasticity. Both routes are again assumed to be identical in terms of cost functions, and it should therefore be no surprise that the optimal first-best tolls are equal for both routes in all cases.

On the left-hand side of Fig. 3, low values for α are found, indicating high demand elasticities. As noted in Section 2, the regulator may in the extreme of a flat demand curve ignore spill-over effects from route T to route U, as road use on route U remains unaffected by changes in f due to

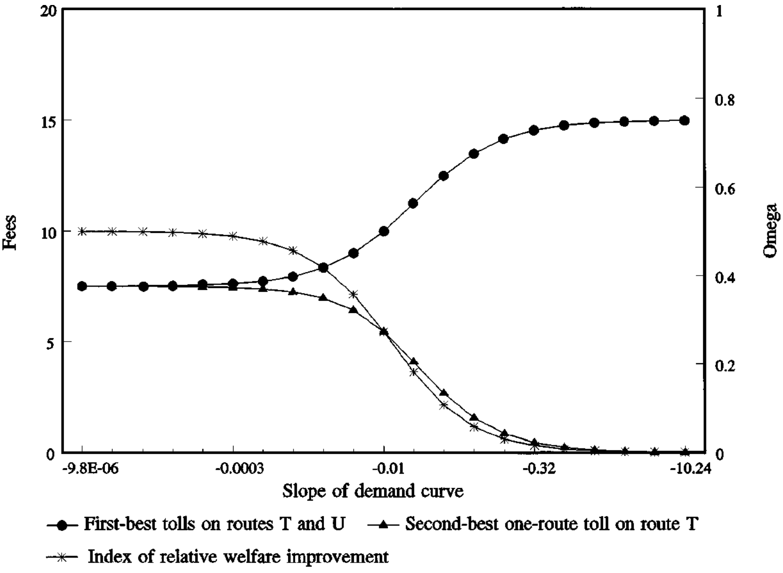

FIG. 3. Varying demand characteristics: optimal fees and index of relative welfare improvement.

the completely homogeneous and sufficiently large group of potential road users. The best thing to do in this case is to optimize usage of route T. This is also reflected by the fact that the optimal one-route fee on route T is equal to the first-best optimal fee. Logically, the index of relative welfare improvement is 0.5 in this case. In the same section however, we asserted that as demand approaches complete inelasticity, regulation should more and more concentrate on route split effects than on overall demand, suggesting increasing scope for one-route tolling. It may in that light seem odd that ω decreases when moving toward more inelastic demand. The reason, however, is that with identical routes, the market itself will take care of optimal route split. As in this case any one-route toll will therefore only be distortionary in this respect; its beneficial effects in terms of overall demand reduction are largely eroded. Put differently, the property of one-route tolling affecting route split is only useful in those cases where the market itself does not lead to efficient route splits, which it actually does when both routes are identical.

As a matter of fact, it is not even enough to introduce a difference in the congestion cost parameter to make one-route tolling only slightly efficient at inelastic demand. As already noted, differentials in congestion cost parameters do not affect the efficiency of a market-based route split.

However, when free-flow user costs differ between the two routes, we obtain the effect suggested in Section 2 where one-route tolling at inelastic demand yields the same welfare improvement as does two-route tolling. In Fig. 4, the tolled route is assumed to have the higher free-flow costs (this case could correspond to two highways between two cities, with the tolled highway being longer than the untolled highway). The optimal one-route toll at perfectly inelastic demand is a subsidy, exactly equal to the difference between the two first-best tolls and yielding exactly the same welfare improvement: $\omega = 1$. As in Fig. 2, the turning point where the optimal one-route toll turns into a subsidy (so that $f = 0$ and $\omega = 0$) marks that specific unfavorable combination of parameters where the sub-goals of route split and overall demand are equally important for overall efficiency but require opposite incentives.

When setting $\kappa_{\mathrm{T}} - \kappa_{\mathrm{U}} = -10$ instead of 10 (that is the case corresponding to the reasonable real-world situation where an arterial road parallel to a toll-road is not priced), optimal one-route taxes (no subsidies) and higher values of ω result throughout. The optimal one-route toll again equals the first-best toll on the tolled route in the limit of completely elastic demand and equals the difference between the two first-best tolls in the limit of perfectly inelastic demand, again with $\omega = 1$. For reasons of space, the corresponding graph is not presented here.

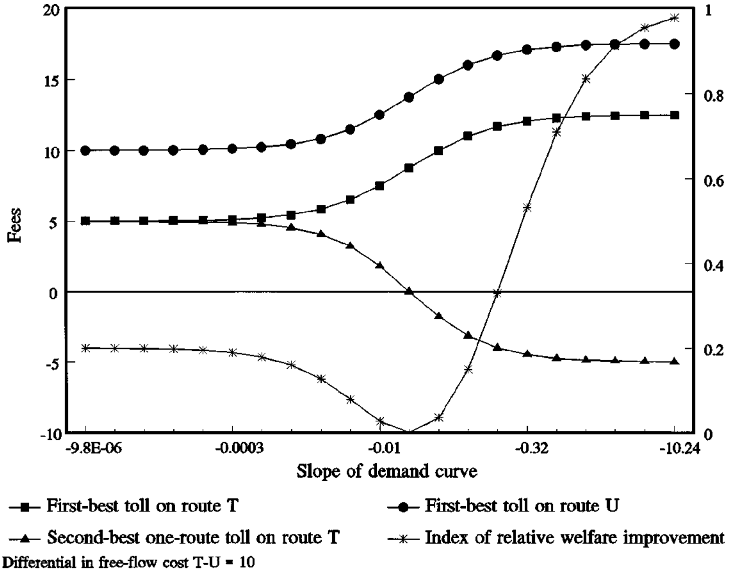

FIG. 4. Varying demand characteristics with a free-flow cost differential: optimal fees and index of relative welfare improvement.

3.4. *Considering the Cost of Congestion Charging*

Obviously, if regulation were costless, there would not be much that could be said in favor of one-route tolling, at least not from an efficiency point of view. In the foregoing sub-sections, we found values of ω generally smaller than 1, indicating a general inferiority of one-route tolling. However, tolling is not costless in practice, and the cost of congestion charging might indeed be such that it is actually preferable to leave one route untolled, as the losses in terms of smaller welfare gains from regulation may be offset by certain savings in terms of the cost of tolling. The simulations presented in the previous section may give an idea of when this could be the case; in other words, when ‘‘rat-running’’ should be allowed from an efficiency point of view. One could distinguish various sorts of costs of congestion charging: (1) a fixed component, depending on the question of whether to regulate at all (for instance, the fixed costs of having a regulatory agency); (2) a route-specific fixed component (for instance, costs of installing electronic devices when using electronic road pricing); and (3) a variable cost component (the cost per regulated user, such as administration costs). As it is these latter two that may eventually determine the choice of whether to leave the option of an untolled alternative, we ignore the first one in what follows.

When considering the cost of tolling within the framework of the simulation model used above, it is not particularly insightful to engage in a "cost-benefit of regulation" type of analysis, in which the policy yielding the highest net benefits would be the most favorable: when dealing with simulated data, one could make any assumption about the costs of regulation. In order then to obtain some insight into the impacts of such costs, let us assume that the same technology is used for both types of regulation, so that the fixed costs of regulation are twice as high for two-route tolling than they are for one-route tolling and the (per vehicle) variable costs of tolling are the same for both policies. We are then able to make at least some general observations on the relative "cost-effectiveness of regulation" of one-route and two-route tolling.

For instance, consider the fixed (route-specific) cost of tolling, assuming that variable (per vehicle) costs of tolling are equal to zero. It can then be asserted that one-route tolling cannot be "overall efficient" (that is, when also taking account of such fixed costs of tolling) whenever $\omega < 1/2$. If $\omega < 1/2$, two-route tolling will yield more than twice as much net benefit, at only double investment costs. Hence, either two-route tolling, or no tolling at all will eventually be the most "overall efficient" option. With $\omega \geq 1/2$, we cannot be sure which type of regulation will be optimal: although the investment in regulation on the second route will yield less net benefits than does the investment on the first route, these additional benefits may or may not exceed the additional cost. A first conclusion then is that when the two routes are identical, the fixed cost of tolling cannot be a reason for choosing on-route tolling: ω for the simulation in Fig. 3 is always smaller than $1/2$, and only approaches $1/2$ when demand tends toward complete elasticity. However, cost differentials between the two routes may indeed cause one-route tolling to be more overall efficient than two-route tolling; in particular when the cost parameters for the tolled route are lower than those for the untolled route so that it is the more important one (see Fig. 2). These are the ranges where it may be inefficient to prevent all rat-running. However, when the cost parameters on the tolled route are higher than on the untolled route, one-route tolling can only be more overall efficient than two-route tolling at relatively inelastic demand (see Fig. 4).

The question of whether the variable (per vehicle) cost of tolling may also affect the relative performance of both types of policy in such a way that one-route tolling may eventually be the more efficient option is not as easy to consider with the simulations presented above, as the inclusion of such variable costs of regulation would actually lead to a new optimization problem, different from (1). Still, a first impression can be obtained by observing that in our simulations, apart from situations with rather inelastic demand in combination with free-flow cost differentials, high values of ω are only found when the tolled route is the relatively attractive one. However, calculations have shown that the welfare improvement per regulated vehicle with one-route tolling in such cases at best only mildly exceeds (and often falls short of) the welfare improvement per regulated vehicle with two-route tolling, which again implies that two-route tolling will often be more overall efficient than one-route tolling, also when accounting for variable cost of regulation. The exception, again, is at inelastic demand with free-flow cost differentials.

In conclusion, our simulations suggest that one-route tolling can in some cases be more overall efficient than two-route tolling, that is, when also considering the costs of regulation. This might particularly occur in situations of inelastic demand in combination with free-flow cost differentials between the two routes and—especially for fixed (route-specific) costs of regulation—when the cost parameters for the untolled route exceed those for the tolled route. Finally, fixed costs of regulation seem to be more of a reason to apply one-route tolling for efficiency reasons than are variable costs of regulation.

4. REVENUE-MAXIMIZING TOLLING UNDER PRIVATE CONTROL

A quite different reason for one-route tolling to occur in practice could be private ownership, with revenue-maximizing tolling on a part of the infrastructure. In this section we will discuss this possibility. We do not consider the investment decisions of such a private owner (which we also ignored with public tolling), and concentrate on the pricing behavior of a revenue-maximizing operator, controlling either a part of the infrastructure or the entire network. We only consider revenue-maximizing non-discriminatory fees: the operator sets just one fee for all users on a route.

4.1. Revenue-Maximizing Fees

The revenue-maximizing one-route fee ϕ can be found by solving the following Lagrangian:

$$\mathscr{L} = \phi \cdot N_{\mathrm{T}} + \lambda_{\mathrm{T}} \cdot \left(D(N) - c_{\mathrm{T}}(N_{\mathrm{T}}) - \phi\right) + \lambda_{\mathrm{U}} \cdot \left(D(N) - c_{\mathrm{U}}(N_{\mathrm{U}})\right) \tag{7}$$

with $N = N_T + N_U$. The first-order conditions are

$$\frac{\partial \mathscr{L}}{\partial N_T} = \phi + \lambda_T \cdot \left(D'(N) - c'_T(N_T)\right) + \lambda_U \cdot D'(N) = 0$$

$$\frac{\partial \mathscr{L}}{\partial N_U} = \lambda_T \cdot D'(N) + \lambda_U \cdot \left(D'(N) - c'_U(N_U)\right) = 0$$

$$\frac{\partial \mathscr{L}}{\partial \phi} = N_T - \lambda_T = 0$$

$$\frac{\partial \mathscr{L}}{\partial \lambda_T} = D(N) - c_T(N_T) - \phi = 0$$

$$\frac{\partial \mathscr{L}}{\partial \lambda_U} = D(N) - c_U(N_U) = 0.$$

Using $\lambda_T = N_T$, we find

$$\phi = N_T \cdot \left(c'_T(N_T) - D'(N)\right) - \lambda_U \cdot D'(N).$$

Solving for λ_U yields

$$\lambda_U = \frac{-N_T \cdot D'(N)}{D'(N) - c'_U(N_U)}.$$

Substitution of λ_U then yields the following revenue-maximizing one-route fee

$$\phi = N_T \cdot c'_T(N_T) - N_T \cdot D'(N) \cdot \left(\frac{c'_U(N_U)}{c'_U(N_U) - D'(N)}\right), \tag{8}$$

whereas it can be shown that revenue-maximizing two-route tolling implies a toll ρ_i on route i ($i = \mathrm{T}, \mathrm{U}$) equal to

$$\rho_i = N_i \cdot c'_i(N_i) - N \cdot D'(N). \tag{9}$$

A first remark on expressions (8) and (9) is that a monopolistic supplier is generally inclined to internalize the congestion externality. This is especially clear in (9): the revenue-maximizing two-route toll consists of the marginal external congestion costs plus a demand-related monopolistic mark-up (see also Rouwendal and Rietveld [14]). Also a comparison of (8) with (2) shows a similarity between f and ϕ: the first term, giving the first-best congestion fee on the tolled route, appears in both expressions.

However, whereas for the optimal second-best one-route toll some non-negative term has to be subtracted, for the revenue-maximizing one-route toll some non-positive term has to be subtracted. Clearly, the revenue-maximizing toll will never be a subsidy. This second term is a variation on the monopolistic mark-up (N in (9) is replaced by N_{T} in (8)), weighted by a factor depending on the slopes of the cost function on the untolled route and of the demand curve. We could trace through these effects in the same way we did in Section 2 for the optimal second-best one-route toll. However, for the sake of space we will not do so but immediately turn to the relative efficiency of revenue-maximizing tolling.

We consider the same basic simulations as discussed in Section 3, and will investigate the index of relative welfare improvement not only for revenue-maximizing tolling on one, but also on both routes (see Table 2 for equilibrium usage in both cases). The reason is that one might suspect that private ownership of both routes may in some instances be preferable to private ownership of one route, due to adverse route split effects in the latter case. Such adverse route split effects may indeed be considerable with revenue-maximizing tolling on one route, as can be seen from the difference between Eqs. (2) and (8): whereas the second term is negative in the former, reflecting the public regulator's concern with spill-over effects, it is positive in the latter, reflecting complete neglect of such effects by a revenue-maximizing operator.

4.2. *Varying Cost Parameters*

Figure 5 shows, for the same free-flow cost parameter differentials as in Fig. 2, the optimal second-best and the revenue-maximizing one-route tolls, as well as the index of relative welfare improvement for these two regimes and for private tolling of both routes. As may be expected, both intuitively and from the expressions for f and ϕ in Eqs. (2) and (8), respectively, the revenue-maximizing one-route toll ϕ exceeds the optimal one-route fee f throughout. The difference between the revenue-maximizing one-route toll ϕ and the optimal one-route fee f turns out to be quite stable, slightly increasing as the free-flow cost on the tolled route becomes higher in comparison to that on the untolled route. In the extreme on the right-hand side, where optimal one-route tolling is in terms of subsidizing usage of route T, the private owner still applies positive tolls. The private toll only falls to zero when non-intervention usage on route T falls to zero, in the most extreme case considered.

Given the relative closeness of the two fees, it will not be surprising that the index of relative welfare improvement for revenue-maximizing one-route tolling (denoted ω_{P}) remains relatively close to ω over the entire range considered, with ω naturally exceeding ω_{P} throughout. It should also be noted that ω_{P} may of course fall below zero when welfare under

TABLE 2

Equilibrium Usage on Both Routes under Revenue-Maximizing One-Route and Two-Route Tolling

	Revenue-maximizing two-route tolling	Revenue-maximizing one-route tolling
N_{T}	$\dfrac{\delta - \dfrac{\alpha\cdot(\kappa_{\mathrm{T}}-\kappa_{\mathrm{U}})}{\beta_{\mathrm{U}}} - \kappa_{\mathrm{T}}}{\dfrac{2\cdot\alpha\cdot\beta_{\mathrm{T}}}{\beta_{\mathrm{U}}} + 2\cdot\alpha + 2\cdot\beta_{\mathrm{T}}}$	$\dfrac{\delta - \dfrac{\alpha\cdot(\kappa_{\mathrm{T}}-\kappa_{\mathrm{U}})}{\beta_{\mathrm{U}}} - \kappa_{\mathrm{T}}}{\dfrac{\alpha\cdot\left(2\cdot\beta_{\mathrm{T}} + (\alpha\cdot\beta_{\mathrm{U}})/(\alpha+\beta_{\mathrm{U}})\right)}{\beta_{\mathrm{U}}} + \alpha + 2\cdot\beta_{\mathrm{T}} + \dfrac{\alpha\cdot\beta_{\mathrm{U}}}{\alpha+\beta_{\mathrm{U}}}}$
N_{U}	$\dfrac{\delta - \dfrac{\alpha\cdot(\kappa_{\mathrm{U}}-\kappa_{\mathrm{T}})}{\beta_{\mathrm{T}}} - \kappa_{\mathrm{U}}}{\dfrac{2\cdot\alpha\cdot\beta_{\mathrm{U}}}{\beta_{\mathrm{T}}} + 2\cdot\alpha + 2\cdot\beta_{\mathrm{U}}}$	$\dfrac{\delta - \dfrac{\alpha\cdot(\kappa_{\mathrm{U}}-\kappa_{\mathrm{T}})}{2\cdot\beta_{\mathrm{T}} + (\alpha\cdot\beta_{\mathrm{U}})/(\alpha+\beta_{\mathrm{U}})} - \kappa_{\mathrm{U}}}{\dfrac{\alpha\cdot\beta_{\mathrm{U}}}{2\cdot\beta_{\mathrm{T}} + (\alpha\cdot\beta_{\mathrm{U}})/(\alpha+\beta_{\mathrm{U}})} + \alpha + \beta_{\mathrm{U}}}$

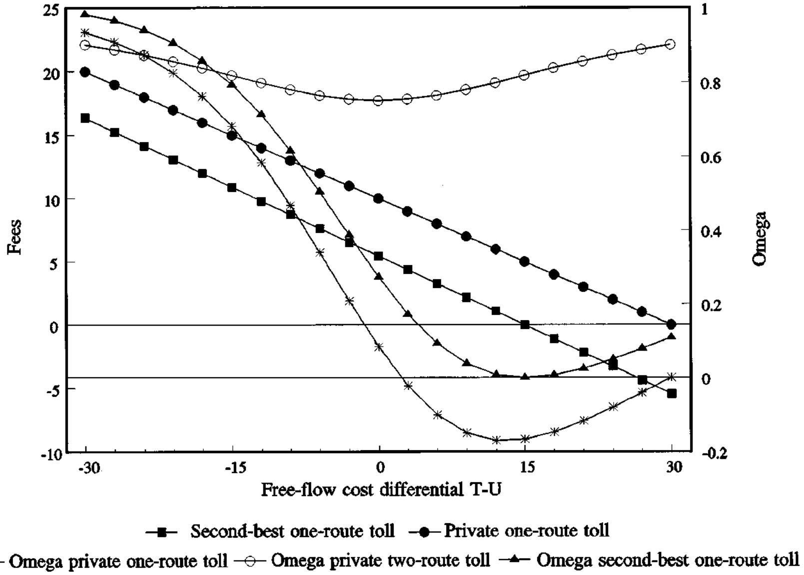

FIG. 5. Varying free-flow costs: optimal and revenue-maximizing one-route fees and indices of relative welfare improvement.

revenue-maximizing regulation is below welfare under non-intervention. For almost the entire right-hand side of Fig. 5 (where free-flow costs on the tolled route exceed those on the untolled route), this indeed is the case.

Next, it is remarkable that the index of relative welfare improvement for revenue-maximizing two-route tolling (denoted ω_{P2}) exceeds ω_P over a considerable range. Apparently, in many cases it is preferable from an efficiency point of view to have a private owner tolling the entire network rather than just a part of it, because in the former case he will have an incentive to avoid adverse route split effects as this will not be in line with overall revenue maximization. Therefore, the intuitive expectation that it is "good" for overall economic efficiency to restrict monopolistic market power does certainly not necessarily hold for the control of congested road networks. Here, it may often be more efficient to have a monopolist controlling the entire network rather than just a part of it, as route split may become seriously distorted in the latter case.

Perhaps even more surprising, private two-route tolling even outperforms optimal one-route tolling over a considerable range, whenever the free-flow costs on the tolled route exceed those on the untolled route and also if the free-flow costs on the untolled route only moderately exceed those on the tolled route. Distortions caused by adverse route split effects, even with optimal one-route tolling, then exceed distortions from monopolistic pricing on both routes. However, a large part of the potential efficiency gain due to monopolistic pricing will of course accrue to the private regulator. Hence, although the effect of monopolistic two-route pricing on overall efficiency of road usage may be attractive, the distribution of these welfare benefits need not be.

For congestion cost parameter differentials (not presented graphically here) ω_P also consistently falls short of ω, the difference increasing the smaller β_T in comparison to β_U (recall from Section 3.2 that these are the ranges where public one-route tolling is relatively attractive). Also, ω_P may again fall below zero, in particular for more extreme congestion cost parameter differentials. Finally, ω_{P2} was again found to exceed both ω_P and ω over significant ranges (this latter especially when the two routes are more comparable in terms of congestion cost parameters).

4.3. *Varying Demand Characteristics*

Finally, Fig. 6 shows what happens when demand characteristics vary. We only present the case with identical routes, as the course of ϕ, ω_P, and ω_{P2} over the range of slopes of the demand curve studied turned out to be hardly influenced by the occurrence of cost differentials on both routes (unlike f and ω; see Section 3.3).

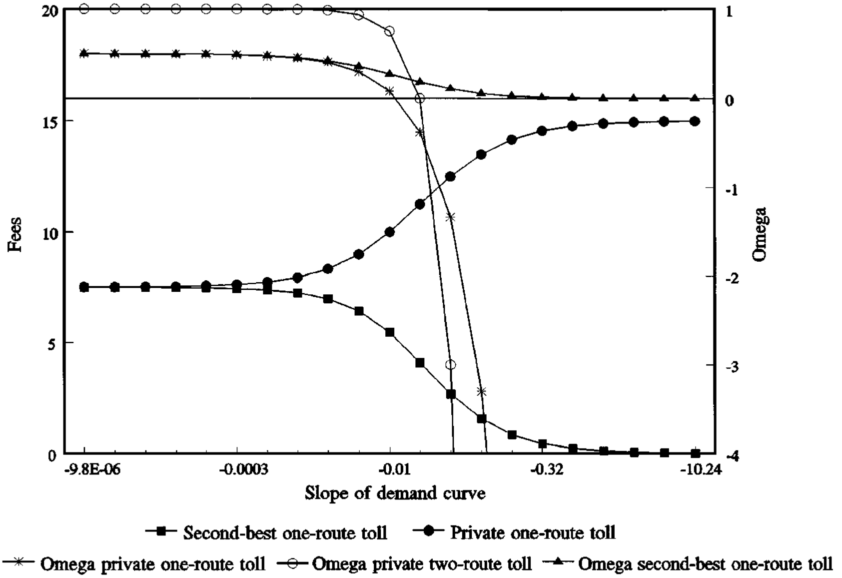

FIG. 6. Varying demand characteristics: optimal and revenue-maximizing one-route fees and indices of relative welfare improvement.

When comparing Eqs. (8) and (2), it can be seen that the two types of one-route tolling will yield a more similar fee, the flatter the demand curve, as then the second terms approach zero and the identical first terms remain. Likewise, when comparing (9) and (3), the same turns out to hold for the two types of two-route fees. This explains the equivalence between f and ϕ, and between ω and ω_{P} on the left-hand side of Fig. 6, as well as the fact that ω_{P2} is practically equal to 1.

On the other hand, as demand becomes more inelastic, the one-route fees f and ϕ rapidly diverge, leading to a fast-growing gap between ω and ω_{P}, with the latter becoming negative due to the fact that it is especially route split (neglected by the revenue maximizer) that becomes important for overall efficiency. The pattern of ω_{P2} is roughly the same, but more extreme. In general, at more inelastic demand, revenue-maximizing tolling tends to become more inefficient due to the extremely high road prices charged. Hence, for the relative performance of revenue-maximizing tolling, be it one-route or two-route, the prevailing demand structure is a crucial factor.

5. CONCLUSION

In this paper we studied second-best congestion pricing in the presence of an untolled alternative. For the various reasons outlined in the introduction, this is certainly not an issue of mere academic interest but one that may be encountered nowadays and in the (near) future in numerous instances. Our findings therefore may be highly relevant for the design of congestion policies.

We considered a two-link network, with one route tolled and one untolled, with elastic demand, taking completely inelastic demand as a limiting case. The second-best one-route congestion toll in its optimal use trades off a number of sub-goals contributing to the overall goal of efficient usage of road infrastructure capacity. These sub-goals are related to usage and congestion on both routes and therefore comprise overall demand and route split effects.

Using a simulation model, we investigated the effects of several parameters relating to the cost and the demand structure on the relative efficiency of one-route tolling. We found that the lower the two cost parameters considered (a free-flow cost parameter and a congestion cost parameter) on the tolled route, the less unattractive one-route tolling becomes from an efficiency point of view. With identical routes, one-route tolling becomes less unattractive the more elastic the demand; when free-flow costs differ between the two routes however, one-route tolling also becomes attractive at inelastic demand, as it is then route split that determines the efficiency of regulation. Concerning the cost of regulation, with fixed (per route) cost of regulation it may indeed be efficient not to toll both routes, in particular if the cost parameters for the untolled route exceed those for the tolled route. Variable (per vehicle) cost of regulation is less of a reason to allow rat-running for efficiency reasons.

Finally, we considered revenue-maximizing tolling on one or on both routes. Revenue-maximizing one-route tolling can by definition never be more efficient than optimal one-route tolling. However, revenue-maximizing two-route tolling may actually lead to a more efficient usage of road space than does optimal one-route tolling. The intuitive expectation that it is ‘‘good’’ for overall economic efficiency to restrict monopolistic power certainly does not necessarily hold for the control of congested road networks. Here, it may often be more efficient to have a monopolist controlling the entire network rather than just a part of it, as route split may become seriously distorted in the latter case. However, when demand becomes more inelastic, revenue-maximizing tolling tends to get more inefficient due to the extremely high road prices charged. Furthermore, a large part of the potential efficiency gain due to monopolistic pricing will of course accrue to the private regulator. Hence, although the effect of monopolistic two-route pricing on the overall efficiency of road usage may be attractive, the distribution of these welfare benefits need not be.

APPENDIX: LIST OF SYMBOLS

N	Total number of road users
N_{T}	Number of road users on the tolled route
N_{U}	Number of road users on the untolled route
$D(N)$	Inverse demand function
$c_{\mathrm{T}}(N_{\mathrm{T}})$	Average social (= marginal private) cost on the tolled route
$c_{\mathrm{U}}(N_{\mathrm{U}})$	Average social (= marginal private) cost on the untolled route
λ	Lagrangian multiplier
r_i	First-best congestion fee for route i
f	Second-best optimal one-route toll
ω	Index of relative welfare improvement
S	Marginal social cost minus marginal benefits
k_i	Marginal social cost on route i
δ	Intersection of the demand curve with the vertical axis
α	(Minus) the slope of the demand curve
κ_i	Average free-flow user cost on route i
β_i	Slope of the average (= private) cost function on route i
ϕ	Revenue-maximizing one-route toll
ρ_i	Revenue-maximizing toll on route i
ω_{P}	Index of relative welfare improvement for revenue-maximizing one-route tolling
ω_{P2}	Index of relative welfare improvement for revenue-maximizing two-route tolling

REFERENCES

1. R. J. Arnott, Unpriced transport congestion, *Journal of Economic Theory*, **21**, 294–316 (1979).
2. R. Arnott, A. de Palma, and R. Lindsey, Departure time and route choice for the morning commute, *Transportation Research*, **24B**, 209–228 (1990).
3. R. Arnott, A. de Palma, and R. Lindsey, Economics of a bottleneck, *Journal of Urban Economics*, **27**, 11–30 (1990).
4. R. Arnott, A. de Palma, and R. Lindsey, A temporal and spatial equilibrium analysis of commuter parking, *Journal of Public Economics*, **45**, 301–335 (1991).
5. R. Arnott, A. de Palma, and R. Lindsey, Route choice with heterogeneous drivers and group-specific congestion costs, *Regional Science and Urban Economics*, **22**, 71–102 (1992).
6. D. Bernstein and I. El Sanhouri, Congestion pricing with an untolled alternative, draft, Dept. of Civil and Environmental Engineering, MIT, Cambridge, MA, 1994.
7. R. M. Braid, Uniform versus peak-load pricing of a bottleneck with elastic demand, *Journal of Urban Economics*, **26**, 320–327 (1989).
8. M. Fujita, ‘‘Urban Economic Theory: Land Use and City Size,’’ Chap. 7.4, Cambridge Univ. Press, Cambridge, UK, 1989.
9. A. Glazer and E. Niskanen, Parking fees and congestion, *Regional Science and Urban Economics*, **22**, 123–132 (1992).
10. F. H. Knight, Some fallacies in the interpretation of social cost, *Quarterly Journal of Economics*, **38**, 582–606 (1924).
11. H. Lévy-Lambert, Tarification des services à qualité variable: Application aux péages de circulation, *Econometrica*, **36**, 564–574 (1968).
12. M. Marchand, A note on optimal tolls in an imperfect environment, *Econometrica*, **36**, 575–581 (1968).
13. E. L. d’Ouville and J. F. McDonald, Optimal road capacity with a suboptimal congestion toll, *Journal of Urban Economics*, **28**, 34–49 (1990).
14. J. Rouwendal and P. Rietveld, Waiting time, monopolistic behaviour and optimality, draft, Dept. of Spatial Economics, Free University, Amsterdam, 1989.
15. D. Starkie, Efficient and politic congestion tolls, *Transportation Research*, **20A**, 169–173 (1986).
16. A. M. Sullivan, Second-best policies for congestion externalities, *Journal of Urban Economics*, **14**, 105–123 (1983).
17. E. T. Verhoef, P. Nijkamp, and P. Rietveld, Second-best regulation of road transport externalities, *Journal of Transport Economics and Policy*, **29**, 147–167 (1995).
18. E. T. Verhoef, P. Nijkamp, and P. Rietveld, The economics of regulatory parking policies, *Transportation Research*, **29A**, 145–156 (1995).
19. E. T. Verhoef, P. Nijkamp, and P. Rietveld, Second-best congestion pricing: The case of an untolled alternative, TRACE discussion paper TI 94-129, Tinbergen Institute, Amsterdam-Rotterdam, 1994.
20. W. S. Vickrey, Congestion theory and transport investment, *American Economic Review*, **59** (Papers and Proceedings), 251–260 (1969).
21. J. Wardrop, Some theoretical aspects of road traffic research, *Proceedings of the Institute of Civil Engineers*, **1**, 325–378 (1952).
22. J. D. Wilson, Optimal road capacity in the presence of unpriced congestion, *Journal of Urban Economics*, **13**, 337–357 (1983).